

of Men; and that it is frequently made use of for the taking of Films from the Eyes of Horses and other Beasts. I hear also that they sell good quantities of it to the Druggists of *London*, but for what use I know not. This is all I could learn concerning *Calamine*. If I have omitted any thing wherein you are desirous of further Information ; or if in any other Concern of this Nature I can be serviceable, you may freely command,

*Wrighton*,  
Oct. 25. 84.

*Tours, &c.*

VIII. *An Arithmetical Paradox, concerning the Chances of Lotteries, by the Honourable Francis Roberts, Esq; Fellow of the R. S.*

**A**S some Truths (like the *Axiomes* of *Geometry* and *Metaphysics*) are self-evident at the first View, so there are others no less certain in their Foundation, that have a very different Aspect, and without a strict and careful Examination rather seem repugnant.

We may find Instances of this kind in most Sciences.

In *Geometry*, That a Body of an infinite Length may yet have but a finite Magnitude.

In *Geography*, That if *Antwerp* be due East to *London*, for that reason *London* cannot be West to *Antwerp*.

In *Astronomy*, That at the *Barbadoes* (and other places between the Line and Tropick) the Sun, part of the Year, comes twice in a Morning to some Points of the Compass.

In

In *Hydrostaticks*, That a hollow Cone (standing upon its Basis) being fill'd with Water, the Water shall press the bottom with three times the Weight, as if the same Water was frozen to Ice; and Figures might be contrived to make it press a hundred times as much.

These Speculations, as they are generally pleasant, so they may also be of good use to warn us of the Mistakes we are liable to, by careless and superficial reasoning.

I shall add one Instance in *Arithmetick*, which perhaps may seem as great a Paradox as any of the former.

There are two Lotteries, at either of which a Gamester paying a Shilling for a Lot or Throw; the First Lottery upon a just Computation of the Odds has 3 to 1 of the Gamester, the Second Lottery but 2 to 1; nevertheless the Gamester has the very same disadvantage (and no more) in playing at the First Lottery as the Second.

It looks very like a Contradiction, that the Disadvantage should be no greater in playing against 3 to 1 than 2 to 1, but it may thus be resolv'd.

Let the  $\left. \begin{matrix} 1\text{ sh.} \\ 2\text{ d.} \end{matrix} \right\}$  Lottery consist of  $\left. \begin{matrix} 3 \\ 4 \end{matrix} \right\}$  Blanks and  $\left. \begin{matrix} 3 \\ 2 \end{matrix} \right\}$  Prizes of  $\left. \begin{matrix} 16 \text{ pence} \\ 2 \text{ shill.} \end{matrix} \right\}$  a piece.

In the first Lottery the Gamester hazards a Shilling to win a Groat, and the Chances being equal, it is evident there is 3 to one against him.

In the Second Lottery the Gamester ventures a Shilling against a Shilling, and the Lots being 4 to 2, his Disadvantage is 2 to 1.

And a Lot at either of them being truly worth just 8 Pence, (*viz.* the 6th part of 3 times 16 Pence, or twice 2 Shillings) the Disadvantage must be the very same

same in both Cafes, that is, the Gamester pays a Shilling for a Lot that is worth but 8 Pence.

The Method of finding this Answer being somewhat out of the common Road, I shall here add it, and thereby infinite Solutions of the same kind may be discovered.

1<sup>st</sup> Lottery.

2<sup>d</sup> Lottery.

Let  $a$  = the number of Blanks.  $m$  = the number of Blanks  
 $b$  = the number of Prizes.  $n$  = the number of Prizes  
 $r$  = the Value of a Prize.  $s$  = the value of a Prize.

$1$  = to what you pay for a Lot, *viz.* a Shilling.

So the Lottery has it's Chances for  $1$ , and the Gamester his for  $r - 1$ . Now the true Odds consisting of the compounded Proportion of the Chances and the Values, *viz.*  $\frac{a}{b}$  and  $\frac{1}{r-1}$ , the Share of the Lottery will be  $a$ , and that of the Gamester  $r b - b$ . Therefore as the present case stands, the first Lottery must be  $a = 3 r b - 3 b$ , and by the like reasoning the second Lottery will be  $m = 2 s n - 2 n$ . Now the Value of a Lot being the Sum of the Prizes divided by the number of Lots, ( which must be equal in both Lotteries ) it yields

$$\frac{r b}{a + b} = \frac{s n}{m + n}$$

So to proceed.

$$\left. \begin{array}{l} a \\ b \\ r \\ m \\ n \\ s \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \right. \left\{ \begin{array}{l} a = 3 r b - 3 b \\ m = 2 s n - 2 n \\ \frac{r b}{a + b} = \frac{s n}{m + n} \\ (*) \\ (*) \\ (*) \end{array} \right.$$

$q = ?$	7	Let $\frac{rb}{a+b} = q$
$7 * \overline{a+b}$	8	$rb = qa + qb$
$8 \times \overline{3}$	9	$3rb = 3qa + 3qb$
$1 + \overline{3b}$	10	$3rb = a + 3b$
9, 10	11	$3qa + 3qb = a + 3b$
<i>Scope</i>	12	If $a = 0$ to avoid negative Numbers.
11, 12	13	$3b = 3qb$
$13 \div \overline{3b}$	14	$q = 1$
12, 14	15	$q > 1$ makes $a < 0$ $q < 1$ makes $a > 0$
<i>Scope</i>	16	If $b = 0$
11, 16	17	$3qa = a$
$17 \div \overline{3a}$	18	$q = \frac{1}{3}$
16, 18	19	$q < \frac{1}{3}$ makes $b < 0$ $q > \frac{1}{3}$ makes $b > 0$
3, 7,	20	$\frac{sn}{m+n} = q$
$20 * \overline{m+n}$	21	$sn = qm + qn$
$21 * \overline{2}$	22	$2sn = 2qm + 2qn$
$2 + \overline{2n}$	23	$2sn = m + 2n$
22, 23	24	$2qm + 2qn = m + 2n$
<i>Scope</i>	25	If $m = 0$
24, 25	26	$2qn = 2n$
$26 \div \overline{2n}$	27	$q = 1$
25, 27	28	$q > 1$ makes $m < 0$ $q < 1$ makes $m > 0$
<i>Scope</i>	29	If $n = 0$
24, 29	30	$2qm = m$
$30 \div \overline{2m}$	31	$q = \frac{1}{2}$
29, 31	32	$q < \frac{1}{2}$ makes $n < 0$ $q > \frac{1}{2}$ makes $n > 0$
15, 19, 28, 32	33	that $a b m n$ may be $> 0$ , $q$ must be $> \frac{1}{2} < 1$

( 681 )

33, 4 (\*)

7, 34.

35 \*, 10

36 -

20, 34.

38 \* -

39 \*  $\frac{2}{-}$

23 \* 3

40, 41

42 -

1  $\div$  37

44  $\div$  3

2.  $\div$  n, 43

46  $\div$  2

5 (\*)

37, 48

45  $\div$  3

6 (\*)

43, 51

47  $\div$  2

34 Let therefore  $Q =$

35  $\frac{rb}{a+b} = \frac{2}{3}$

36  $3rb = 2a + 2b = a + 3b$

37  $a = b$

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38  $\frac{sn}{m+n} = \frac{2}{3}$

39  $3sn = 2m + 2n$

40  $6sn = 4m + 4n$

41  $6sn = 3m + 6n$

42  $4m + 4n = 3m + 6n$

43  $m = 2n$

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44  $1 = 3r - 3$

45  $3r = 4$

46  $2 = 2s - 2$

47  $2s = 4$

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48 Let  $A = 3$

49  $B = 3$

50  $R = \frac{4}{3}$ , id est, 16 Pence.

51 Let  $M = 4$

52  $N = 2$

53  $S = 2$  2 Shillings.